

Evaluating cumulative ascent: Mountain biking meets Mandelbrot

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Abstract.

The problem of determining total distance ascended during a mountain bike trip is addressed. Altitude measurements are obtained from GPS receivers utilizing both GPS-based and barometric altitude data, with data averaging used to reduce fluctuations. The estimation process is sensitive to the degree of averaging, and is related to the well-known question of determining coastline length. Barometric-based measurements prove more reliable, due to their insensitivity to GPS altitude fluctuations.

Keywords: GPS, altitude measurement, Mandelbrot, fractals

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1. Introduction

The typical mountain bike ride includes changes of altitude, and the effort expended depends, in part, on the overall distance climbed. Examples of other contributions to overall effort, some neither quantifiable nor reproducible, include riding speed, terrain conditions and weather, each of which can vary considerably during a ride. The total vertical ascent is an apparently simple characteristic of a given route, so that it would be appropriate to assign it a numerical value. It turns out that this is a more difficult task than anticipated, with the measurement process itself not necessarily able to provide a unique answer.

The problem is analogous to the well-known exercise of measuring the length of the coastline of a country, with Great Britain the instance headlined by the work of Richardson and Mandelbrot [1,2]. Length estimates can vary substantially, depending on the scale of the map used, because smaller coastline features appear as resolution is increased, an observation that leads to the consideration of irregular forms in general. For the overachieving cyclist, however, estimating cumulative ascent is an important practical matter. While the late Benoit Mandelbrot is not reported to have encountered mountain biking in person, the methodology spawned by adopting a fractal perspective when taking the measure of geometrically irregular shapes can offer guidance in addressing the question.

Modern technology, in the form of the Global Positioning System (GPS) satellite constellation, and affordable, compact GPS receivers, provides the necessary information. A GPS receiver acquires data from multiple satellites, which it processes to supply the user with a relatively precise geographical location and an altitude estimate. Location is expressed either as latitude and longitude or, after conversion, as map grid coordinates, while altitude is measured relative to a model of the earth’s essentially ellipsoidal geoid. Positional accuracy depends on the locations of the satellites offering the strongest signals, as well as on local topography and other obstructions that might degrade signal quality.

An alternative means of altitude measurement relies on the height variation of atmospheric pressure. A barometric device, after suitable calibration, can be used to measure height above a reference level, assuming no pressure changes due to meteorological causes. Certain GPS receivers incorporate a barometric altimeter that is continually recalibrated using GPS data, but which, due to a relatively slow response, is far less susceptible to varying GPS signal quality and thus provides more stable results.

The present paper describes the outcome of tests using both types of altitude measurement. Different degrees of data averaging are applied to reduce the effect of the measurement ‘noise’. It is apparent from the results that there is no preferred length scale in the problem that could help determine the optimal averaging, leading to the somewhat unexpected conclusion that there is really no correct answer to the question, the same as reached when attempting to measure the length of the British coastline.

2. Methods

Two GPS receivers were mounted on the same bicycle to ensure they followed a common space-time trajectory. One was the Garmin Edge 205, a receiver intended for cycling use that employs GPS-based altitude measurement, while the other was the Garmin Dakota 20, a more general-purpose device that also incorporates a barometric altimeter [3]. Each receiver records its position history at a variable rate designed to allow reproduction of an accurate track, ideally of a quality suitable for subsequent almost map-free navigation. The history consists of a series of trackpoints, each specifying time of day, horizontal position expressed as latitude and longitude, and altitude; it can be retrieved for computer processing, in the case of the Edge using, e.g., the open source GPSTracker [4] software, and for the Dakota by copying the GPX-format data file. Trackpoints are converted to map coordinates with the appropriate transformation functions [5] – the relevant conversion for the present work being the Cassini-Soldner projection – borrowed from the GPSTracker source code; horizontal distance measurements then follow immediately.

The hilly terrain conditions over the first of the closed routes that provided data for analysis are typical for offroad cycling, with partial tree cover and topography capable of degrading GPS signal quality; results from two other trips, one involving similarly

hilly conditions and the other over flatter terrain, are discussed briefly towards the end of the paper. Good horizontal positional accuracy is achieved over the 42 km (measured horizontally) track, and the tracks from the two receivers are essentially identical when superimposed on a 1:50000 scale map. While there are many sources of error, ranging from suboptimal GPS signal reception to the analog-to-digital conversion of the barometric readings, no additional information, e.g., details of the proprietary algorithms used to process the raw GPS data, is readily available that could be utilized in the analysis.

The methods used by the receivers for determining recording rate are not specified, but the measured time-interval distribution for the Edge (~ 2280 points recorded) has a sharp peak at 6.5s and a small peak at 1s, with essentially all intervals < 13 s (longer intervals correspond to a nominally stationary receiver), while for the Dakota (~ 1600 points) there is a peak at 12s and a weakly split peak in the range 1-4s with most intervals < 20 s; the time intervals show no obvious speed dependence (those above 20s only appear at speeds below 5 km/h). The corresponding distance-interval distribution for the Edge peaks at 20m, while the Dakota has a peak at 8m and a weakly split peak over 20-40m. The maximum distance interval increases with speed, and for any given speed there is a broad spread of values; the maxima (excluding a few outlying points) at the overall mean speed (12 km/h) are 40m for the Edge and 60m for the Dakota. Larger intervals might affect distance accuracy if there are sharp changes in direction, although these become less likely at higher speeds (and direction changes may even reduce the interval size, another unknown aspect of receiver behavior). Variable intervals allow much longer track histories to be recorded, compared to the use of a fixed and necessarily short interval, but their use can complicate the altitude analysis.

3. Results

Altitude data measured by the two GPS receivers over the duration of the first trip are shown in Fig. 1. The two sets of data overlap reasonably well, after allowing for a 0.4 km (1%) difference between the horizontal track length estimates, that of the Edge being longer, with the difference probably due to the fact that 40% more trackpoints are recorded. However, the GPS-based altitude data from the Edge is considerably more noisy than the barometric Dakota; an example is shown in Fig. 2. Altitude measurements based only on the GPS signal are susceptible to spikes and other irregularities due to momentary signal degradation, e.g., the prominent spike in Fig. 1 at 4 km associated with signal loss while in a tunnel under a highway; averaging will be essential to reduce their otherwise serious impact on cumulative ascent evaluation. The altitude varies between 260m and 800m over the route, so while this 540m difference represents a lower bound for the total ascent, the question is how much extra climbing is actually present in the data?

The spurious contributions of fluctuating altitude measurements can be reduced by averaging. For the initial attempt at analysis, symmetric equally-weighted averages are

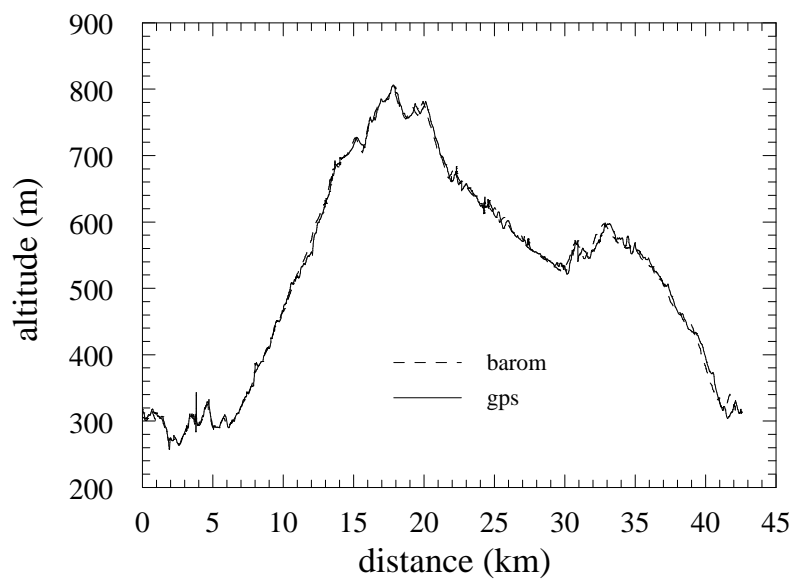


Figure 1. Altitude estimates vs distance obtained from the Garmin Dakota (barometric data) and Edge (GPS data) receivers.

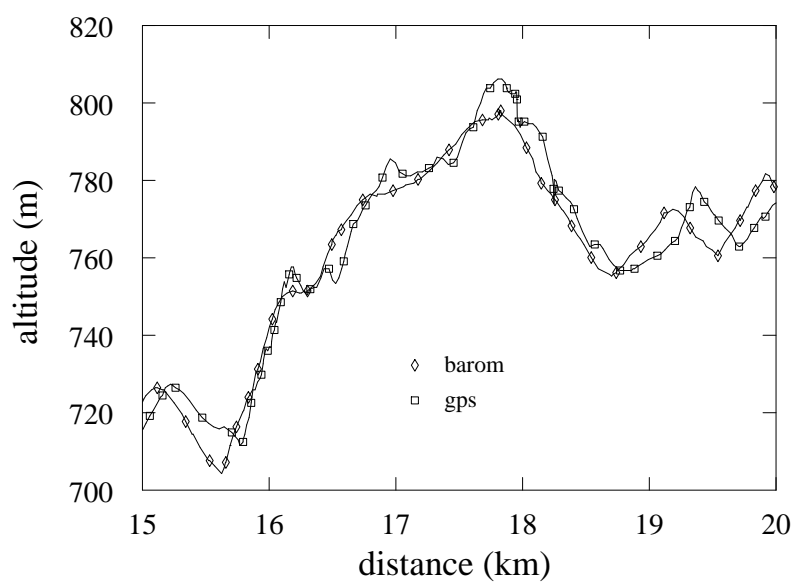


Figure 2. Enlarged portion of Fig. 1 showing the differences in altitude data (symbols every 5th point).

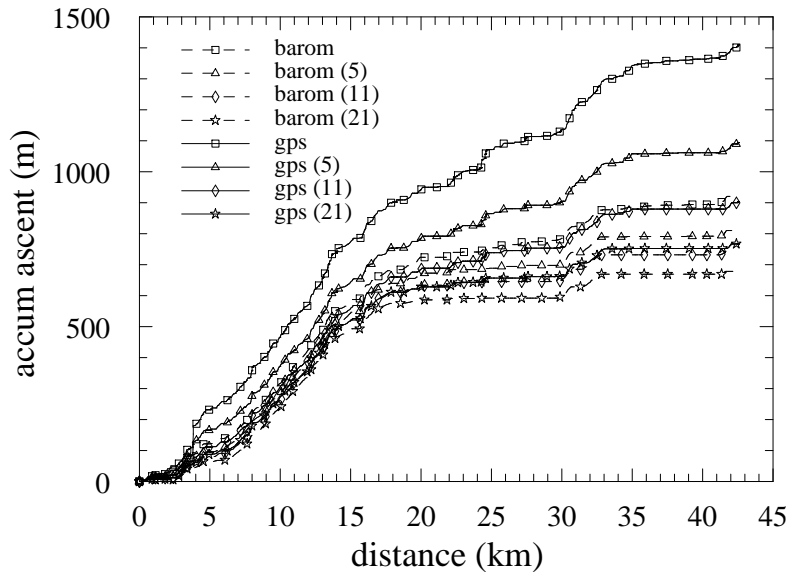


Figure 3. Accumulated ascent vs distance from the data of Fig. 1; the original data and averages over specified numbers of points are shown (symbols every 50th point).

evaluated about each data point, where the number of points included is regarded as a parameter. The cumulative ascent is then computed along the track. Results based on averages over 5, 11 and 21 successive points, as well as on the unaveraged data, are shown in Fig. 3. The terminal value of each curve represents an estimate of the total ascent. These are seen to range from $\sim 680\text{m}$ to 1410m , and while the largest values are clearly gross overestimates, and the smallest underestimates, the remaining values, all seemingly plausible, are spread over a relatively broad $\sim 200\text{m}$. For a given degree of averaging, estimates based on GPS altitudes are all substantially larger than the barometric results, consistent with the increased fluctuations noted previously.

A systematic examination of the dependence of total cumulative ascent $C_a(s)$ on s , the number of points used in the averaging – itself only a rough measure since the distance between consecutive trackpoints is not fixed – is carried out with the aid of a log-log plot. This is a widely-used technique for analyzing data from processes considered to be devoid of intrinsic size scales, and the result appears in Fig. 4. The absence of a plateau is a signature of the scale-free nature of the problem. Reasonably good fits of the power law expression [2] $C_a(s) = Gs^{(1-D)}$ to the plotted data, over approximately a single decade, lead to exponents $D = 1.26$ and 1.18 for the GPS and barometric altitudes respectively, but see [6,7] for a discussion of how many decades are needed to establish ‘fractality’. Unfortunately, such an outcome is of limited interest, given that the quantity s does not correspond directly, but only on average, to a physical distance.

A more meaningful approach to reducing the effects of fluctuations takes the real

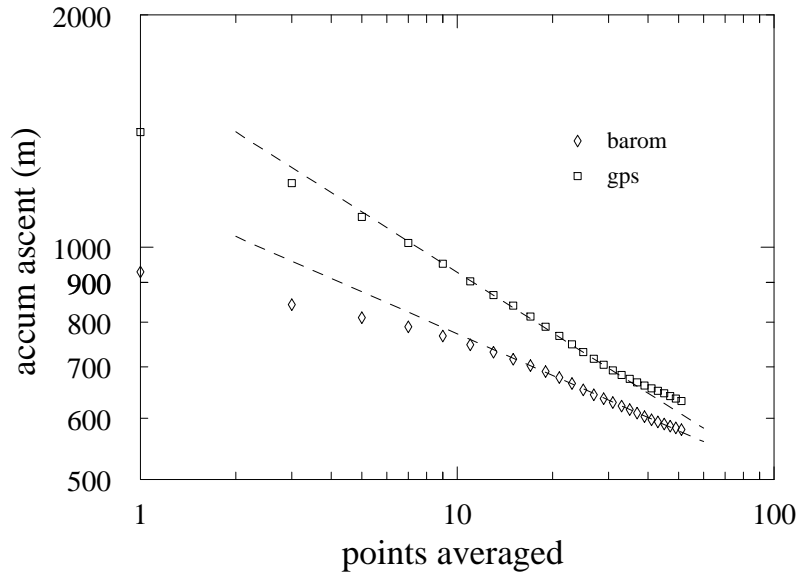


Figure 4. Log-log plots of total accumulated ascent based on averaging over different numbers of points from the data of Fig. 1, together with power-law fits (see text).

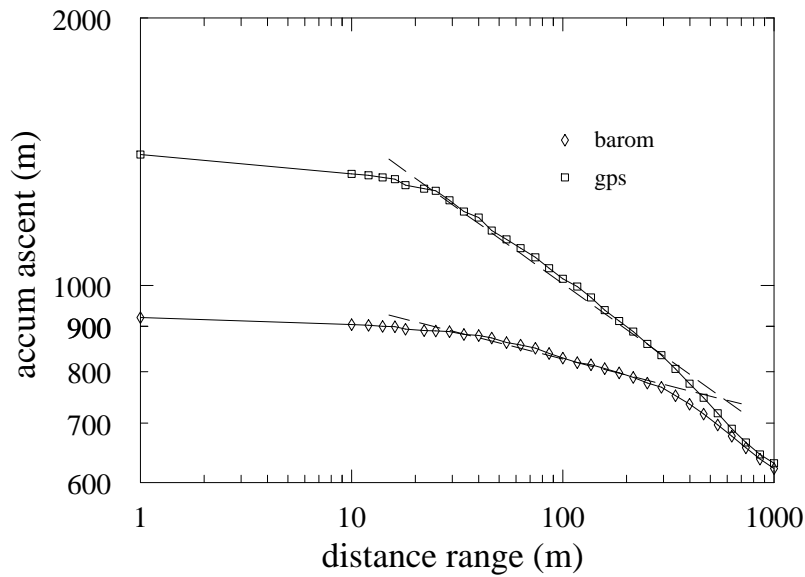


Figure 5. Log-log plots of total accumulated ascent based on averaging using a distance window applied to the data of Fig. 1, for different distance ranges; power-law fits are included (see text).

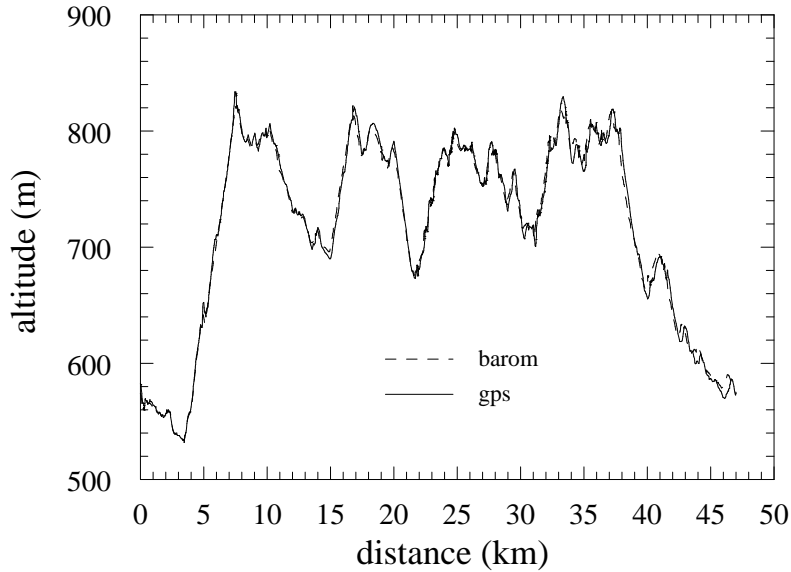


Figure 6. Altitude estimates vs distance from the two receivers over the second hilly route.

separation of successive trackpoints into consideration, with a sliding distance window used to select the series of neighboring points that contribute to the averaged altitude at each data point. Although additional computation is involved, and the method is unsuitable for real-time usage since the averaging is symmetric, it corresponds to the methods used for coastline measurement [2]. Fig. 5 shows a log-log plot of the results obtained using windows spanning different distance ranges. Power-law fits over the near-linear 30-300m range yield exponents $D = 1.17$ and 1.06 for the GPS and barometric altitudes, with the larger value, as before, a consequence of the GPS-based altitude fluctuations.

In view of typical terrain conditions, the ~ 20 -50m window range is likely to be the most relevant. Over this range the barometric data yields a spread of ascent values between 860m and 890m, and in the absence of averaging the value increases slightly to 920m, so the uncertainty amounts to $\sim 4\%$. The much larger GPS-based values are seen to need averaging over almost 200m on average before reaching even the unaveraged barometric estimate, with a 200m wider data window required for similar estimates at intermediate ranges, e.g., 300m vs 100m; this essentially fictitious contribution to the cumulative ascent should grow linearly with the time spent in motion. At widths beyond 700m both sets of estimates are essentially the same, suggesting that GPS-based averages over such large distance ranges are no longer sensitive to the measurement fluctuations; however, excessive averaging will erase real topographical features, such as intermediate hilltops, leading to an underestimated total ascent.

Similar analysis was carried out for the track whose altitude measurements appear

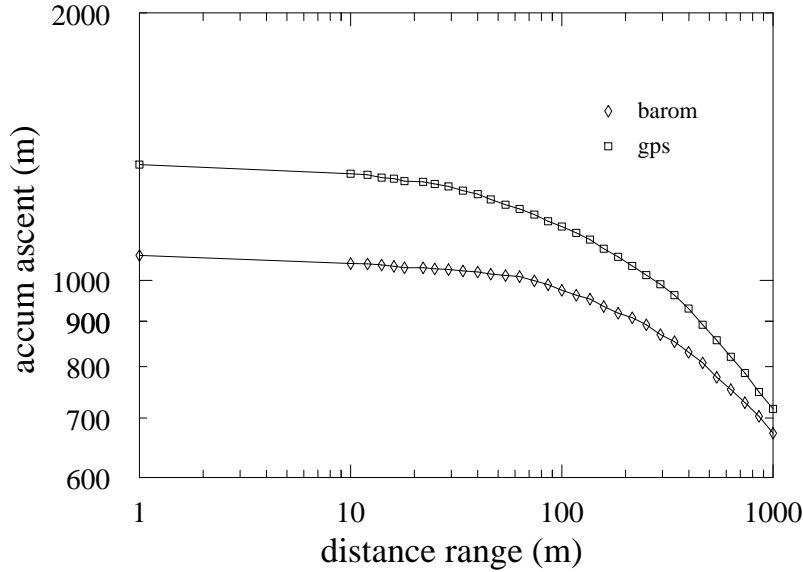


Figure 7. Total accumulated ascent based on averaging using a distance window applied to Fig. 6.

in Fig. 6 (track length 47 km, with 1640 and 2210 trackpoints recorded); although difficult to see at this scale, the altitude plots are subject to the same kinds of small differences as in Fig. 1. Accumulated ascent estimates using distance-based averaging are shown in Fig. 7. The barometric measurements using 20-50m averaging windows are in the range ~ 1010 - 1030 m, increasing to 1070 m without averaging. The GPS-based results overestimate ascent as before, with averaging over at least ~ 200 m again needed to recover the barometric estimates. (The near-linear range available for a power law fit is reduced to less than a decade, with fits over the 50-200m range yielding $D = 1.13$ and 1.10 , values within a few percent of the previous track; note that coastline measurements also do not indicate D to be universal.)

The final example involves a track free of significant altitude changes (length 44 km, 1240 and 1850 trackpoints recorded). Altitude measurements are shown in Fig. 8, where the GPS-based results are again subject to larger fluctuations than the barometric values. The fact that the fluctuations appear more prominent is due to the greatly reduced altitude range, now only 60m. While in the previous cases it might have been possible to make rough visual estimates of the total ascent, here the result is less apparent.

The accumulated ascent estimates using distance-based averaging are shown in Fig. 9. For this track, the GPS-based results overestimate ascent by a substantial factor, rendering them useless in practice. The barometric measurements are more reasonable, with the 20-50m averaging range again providing a good basis for estimation. The total

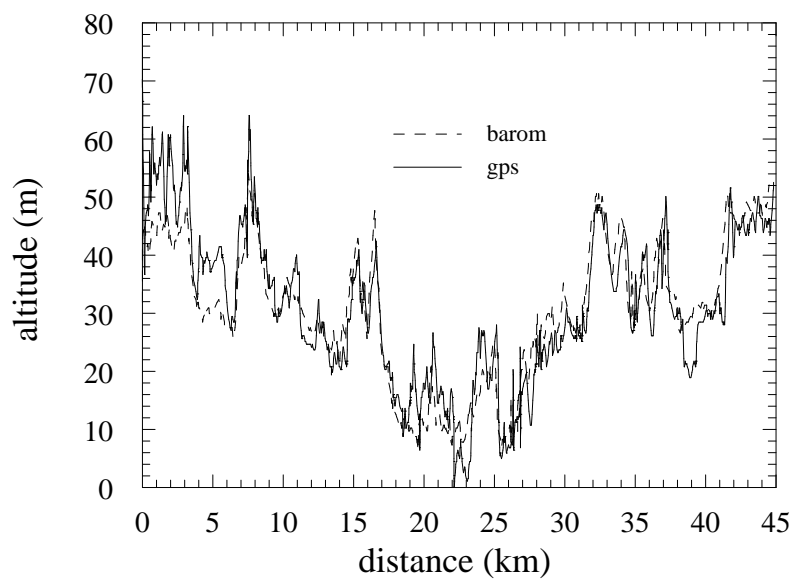


Figure 8. Altitude estimates vs distance for a relatively flat route.

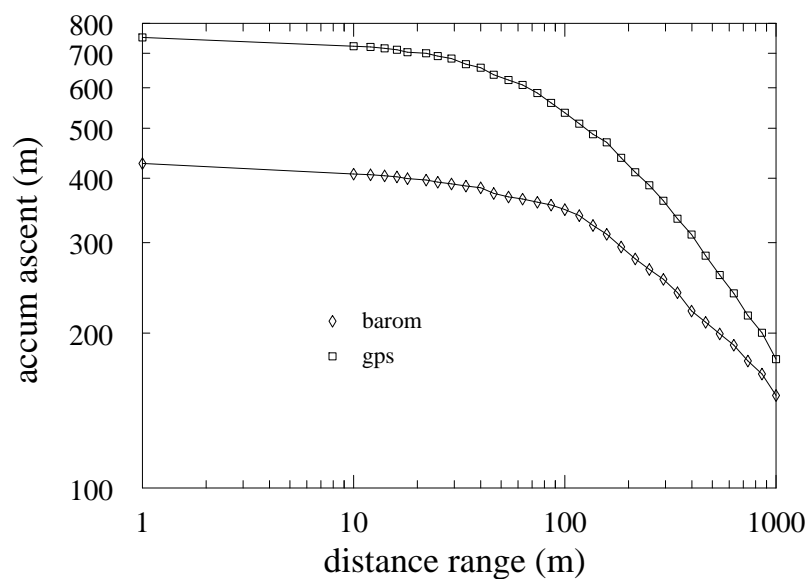


Figure 9. Total accumulated ascent from the data of Fig. 8.

ascent is in the range ~ 370 - 400 m, a similar spread as before (but now amounting to 8%), while omitting the averaging adds another 30m; larger windows again tend to erase real terrain features. (In the case of the GPS-based altitudes, exponent estimation is not possible, whereas the barometric altitudes apparently exhibit two distinct subranges below and above the 100m value, with exponents $D = 1.09$ and 1.35 . Averaging based on fixed numbers of data points leads to $D = 1.63$ and 1.44 , values larger than for the first track that reflect the enhanced contribution of short-range fluctuations in the data.)

Determining the extent to which observations of this kind are generally applicable requires more extensive experimentation covering a wider selection of topographical conditions, although the trend seems clear. It should also be pointed out that while averaging tends to eliminate small-scale features that might be little more than ‘humps’ along the trail, not all such humps are equal; a small ascent followed by the corresponding descent might go almost unnoticed when embedded in a horizontal or downhill segment, simply due to momentum, but if superimposed on an already significant uphill grade its presence will certainly be felt. Thus, even a well-defined measure of ascent would be unable to characterize completely its contribution to the overall effort.

4. Conclusion

The outcome of this analysis, which is not offered as a systematic study of the problem but merely as a limited set of observations, is the conclusion that estimating cumulative ascent, an important measure of effort expended by a cross-country cyclist, is an ill-defined task. At best, a range of estimates can be obtained, hopefully one that is comparatively narrow. Furthermore, the use of barometric altitude data appears essential for meaningful results; GPS-based altitude data, without a substantial degree of averaging, leads to overestimation. The question as to which particular estimate is the ‘correct’ one, just as when measuring the length of the coast of Britain (or almost any other country, except Nauru), has no unique answer; an approach to the data analysis based on that used for studying fractal-like phenomena helps clarify the situation. While the coastline problem might be more of an intellectual curiosity, cumulative ascent – at least to some – is of considerable practical importance.

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